SMART WATER MANAGEMENT PLATFORM

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WP3

D3.2 Optimization for Water Irrigation

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2	Intercrop	ICRO	ES
3	University of Bologna	UBO	IT
4	Consorzio di Bonifica dell'Emilia Centrale	CBEC	IT
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Abbreviations

ACO	Ant Colony Optimisation
CBEC	Consorzio di Bonifica Emilia Centrale (Italian pilot)
CP	Center Pivot
DI	Deficit Irrigation
ET	Evapotranspiration
FAO	Food and Agriculture Organization of the United Nations
GA	Genetic Algorithm
GRASP	Greedy randomized adaptive search procedure
GRASP	Greedy Randomized Adaptive Search Procedure
ICT	Information and Communication Technologies
IoT	Internet of Things
LP	Linear Programming
MPC	Model Predictive Control
MZ	Management Zone
RDI	Regulated Deficit Irrigation
VRI	Variable Rate Irrigation

Executive Summary

This deliverable describes the SWAMP approach for Irrigation Planning (Optimisation), as part of the Work Package 3 - Estimation and Optimisation Services. The Water Need models (D3.1) provide the water demand estimates for the fields based on soil, plant and weather information. The Irrigation Optimisation component, on the other hand, is concerned with how to plan the irrigation itself in terms of water allocation and irrigation timing. It has to find a plan that achieves the pilots' objectives while ensuring that high-level operational constrains of the farms are met. Such constraints include operational costs with water and energy, limits in water supply and of the irrigation methods employed. A review of the literature in irrigation scheduling is presented, along with its limitations for the SWAMP scenarios. The planning problem is described first in general terms and then formally. Two variations of the problem are identified based on the characteristics of the SWAMP pilots: the general (or modular) case, where fields can be irrigated semi-independently (subject to constraints on shared elements of the system); and the Center Pivot case, where several management zones in a circular field are irrigated by a moving array of sprinklers. Linear and nonlinear programming models for tackling those problems are presented, as well as a discussion on the use of metaheuristics for dealing with their computational complexity.

1. Introduction

The SWAMP project develops IoT based approaches for smart water management in precision irrigation domain and pilots these approaches in both Europe and Brazil. The present document is an output of Task 3.2 (Optimisation of irrigation analysis), within Work Package 3 (Estimation and Optimisation Services). It describes the SWAMP approach to Irrigation Planning and refers to other deliverables in WP3 (D3.1 - Water Need Models and D3.2 - Water Distribution Optimisation), as well as other project deliverables.

Throughout this document, Planning and Scheduling will be used interchangeably, meaning the decision of how much and when to irrigate the field(s). Optimisation is the means by which planning is achieved, and it is subject to the objectives and constraints of the SWAMP pilots.

1.1. Scope and Motivation

This deliverable is related to various other deliverables and summarizes their contents where needed. Particularly, deliverables D3.1 Water Need estimation [28], and D5.1 Pilot Specification [18] are cited and must be used as the source whenever further information is required. The relationship with deliverable D3.1 and the upcoming D3.3 Water Distribution Service is illustrated in Figure 1. The Irrigation Optimisation component also consumes data from core platform services, as described in D1.2 Initial SWAMP Core Platform [14].

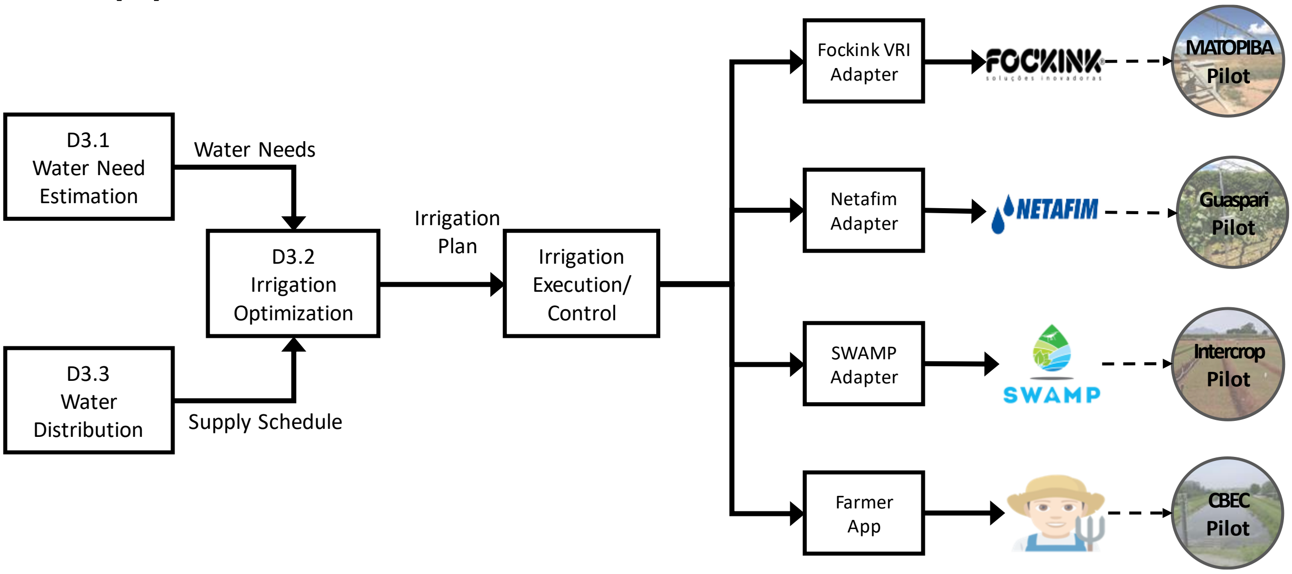

FIGURE 1 RELATIONSHIP WITH OTHER SWAMP SERVICES.

The Water Need Estimation methods described in D3.1 address the question of how much water the crops demand in the near future in order for the plants to grow healthy. They use weather, soil and crop data collected from probes and drones, and combine physical and data-driven models to compute these estimates. These methods provide what can be called the Ideal Irrigation, i.e., the amount of water that ideally should be replenished in the soil to compensate for the evapotranspiration and maximize production. There are, however, other aspects that need to be considered by an actual irrigation plan, i.e., a plan that can be put in place considering farm and crop characteristics, pilot requirements and constraints, and at the same time providing the best economic return for the farmer. Examples of such aspects:

- **Water availability**: water scarcity is a problem in various parts of the planet. In the Intercrop Pilot (Cartagena), for example, part of the water comes from desalinization plants, and droughts can affect water availability throughout the year. Another scenario is the existence of water quotas or supply schedules. In the Italian pilot, farmers have to request water to a Water Distribution Consortium, which manages the channel network that delivers water to the farms. The Water Distribution service (upcoming D3.3) will be responsible for handling these requests. In case when not enough water is available, the irrigation plan should allocate it according to their economic return to farmer.

- **Costs of irrigation:** even if the water comes from private reservoirs, irrigation is not free. Pumping the water to the fields consumes energy, whose cost have an impact in the farmer's bottom-line. For example, in the MATOPIBA pilot's farm, the energy bill can account for up to 30% of the production cost. In order to reduce this figure, irrigation there often happens during the night, which has a 90% decrease in the tariff.

- **Limitation of the irrigation systems:** different irrigation methods deliver water to the plants with varying efficiency. According to the FAO Irrigation Manual 4 [5], Annex I, furrow irrigation presents heavy loses to the ground from percolation and run-off, resulting in 60% efficiency. Sprinkler irrigation is more efficient, at 75%, with loses through evaporation for example. Drip irrigation is the most efficient of the more commonly used methods, at 90%, since it delivers water slowly and very close to the plants' roots. Other aspects of the irrigation system should also be considered: maximum pumping capacity of the farm, minimum and maximum irrigation volumes per field, uniformity of irrigation over fields with spatial soil variability, among others. These constraints can limit the number of fields that can be irrigated at the same time, for example. In Center Pivot fields, the time to complete a full turn must be considered when scheduling the irrigation of each Management Zone. Although the SWAMP project is aimed at IoT-enabled, fully automated irrigation, there might be at least parts of the system that have to be operated manually. For example, irrigation cannot be scheduled for the middle of the night in case we need an employee to open the valves. These general constraints should be considered in order to avoid creating an irrigation plan that is not feasible in practice.

Given the described above, the goal of Irrigation Optimisation is to compute a schedule of irrigation events that best addresses the water needs of the crops while being aware of high-level operational constraints and the economic interests of the farmer. The plan serves as input to the Irrigation Execution and Control service shown in Figure 1. This service communicates with the software modules running in the farms via special adapters. These software modules control the opening and closing of valves, the pressure at pumps, and so on.

1.2. Deliverable structure

The remainder of this document is organized into the following sections.

- Section 2 (Background on Irrigation Scheduling) describes briefly the traditional practices in irrigation planning and other relevant works in the literature.
- Section 3 (Problem Description) presents the planning problem in general terms, along with the assumptions and specificities for the SWAMP pilots.
- Section 4 (Modelling and Solutions) formalizes the problem and proposes strategies for tackling it.
- Section 5 (Discussion) brings considerations about the solutions and well as possible improvements.
- Section 6 (Conclusions) presents the final thoughts and next steps for addressing the problem.

2. Background on Irrigation Scheduling

The traditional approach for irrigation planning, as seen for example in the FAO Training Manual 4 [5], considers the entire irrigation season. In this approach, the crop water needs are estimated for its various development stages (using evapotranspiration methods for example), considering the average weather conditions in the region for the season. The total water amount is then spread across the season through discrete irrigation events. In order to simplify the logistics of irrigation for the farmers, the same volume is used in all events, which are evenly spaced through the season. A more sophisticated approach uses different water amounts or number of irrigation events depending on the crop development stage (FAO manuals define 4 different stages: initial, crop development, mid-season and late season). The soil type and its effects on water absorption and retention are also considered in the calculations, since the soil acts as a buffer for the water.

The traditional scheme has the advantage of a low entry barrier, since it relies on historical averages and well-known estimates for crop water needs. The simplicity of the irrigation plans is also an advantage, since it allows farmers to execute them more precisely. The clear limitation is that planning with months in advance might incur in large estimation errors. In the end the farmer will have to adjust the plan using his/her experience, postponing or anticipating irrigation events to compensate for the actual precipitation, for example.

Improving on the traditional approach, some works try to optimize the water and crop allocation and scheduling over an entire irrigation district comprised of multiple farms. The work in [1] proposes a multilevel approach for maximizing the expected annual revenue yield in a scenario where crops compete for both irrigation water and cultivation area. The authors consider uncertainty of year-long predictions using a multi-layered stochastic dynamic programming approach. The goal is providing optimal irrigation water allocations over the growing season to each crop individually, considering limitations of the total available water. They obtained significative gains of 49% or more, compared to the traditional methods used in their test scenario in Iran.

Other works focus on minimizing the overall costs with pumping by saving electricity or selecting optimal tariffs. Reca et al. [24], for example, propose an optimal pumping scheduling scheme for a pressurized water distribution system in Spain. Using linear programming they achieved up to 24% in savings with energy. Perea et al. [22] used Genetic Algorithms for grouping hydrants and water allocation among farmers in a pressurized network. Their optimisation solution considered soil water balance to avoid stressing the crops, obtaining savings of 14% in energy cost, with marginal gains in yield.

In common, these approaches have to rely on historical patterns of water usage by farmers and weather forecasts for the whole irrigation seasons (several months), which affects their accuracy. Considering that, Corceles et al. [9] focus on optimizing irrigation events for on-demand irrigation in a pressurized network in Spain. Their goal is to schedule the opening of hydrants to optimize energy costs, which they manage to reduce by 10% using simulations in Matlab. Similarly, the work in [15] also uses Matlab simulations to minimize irrigation costs with pumping. The authors computed weekly schedules considering the energy tariffs offered by different providers for the entire irrigation district. In a follow-up paper [16], they showed the effects of the scheduling when combined with regulated deficit irrigation (RDI), which reduced the water requirements and provided savings with energy of more than 24%.

The works described above try to optimize the irrigation scheduling for multiple farms in the same irrigation district. SWAMP, however, separates water distribution and the final irrigation scheduling at the farm, giving the farmer control over his/her property. For on-farm irrigation scheduling, Brown et al. [6] try to maximize all-season profit using Simulated Annealing for computing irrigation strategies, where strategies consist of soil-moisture thresholds for triggering irrigation (however, moisture was estimated, not measured as in SWAMP). For dealing with the uncertainty of season-long optimisation, a stochastic model of the weather was used. Their approach provided average gains in profit of around 9%. Pham et al. [23], in their turn, used an iterative algorithm coupled with FAO's AquaCrop [27] water productivity simulator to reduce water usage without affecting productivity. Their results showed 12% savings in water for a year-long season, with marginal gains in yield.

An Ant Colony Optimisation (ACO) algorithm was used in [21] for optimizing water irrigation and fertilizer application in a Center Pivot field. The optimisation considered the seasonal rate of available irrigation water (ranging from 22% to 100%), using the RZWQM2 growth simulator for measuring the effects on crops. Results for a three-year simulation period showed 14.7% and 16.7% water and fertilizer economy, respectively, with similar net return.

The works above try to optimize the irrigation schedule for a single farm, but the whole season, therefore having to deal with the inaccuracy of long-term predictions, especially weather forecast subject to climate change, as noted before. Saleem et al. [25] propose a strategy for on-farm, short-term irrigation scheduling, later extended in [10]. In the former, on-demand irrigation is optimized using Model Predictive Control (MPC), which is used to adjust soil-moisture in real-time and minimize plant stress. The later enhances the original MPC controller to consider water availability when scheduling the irrigation. It also includes a weather forecast model with uncertainty. Their results show significant gains in water economy (up to 50%) against fixed-depth schedules when water is plentiful, but marginal gains when scarcity is a problem. In a similar vein, the works by Winkler, Cerpa and colleagues [31], [32], focus on soil-water models and sensors spread across the turf field to minimize water consumption by 24%.

The SWAMP approach shares some similarities with the previous works as it also focuses on near real-time scheduling of irrigation based on data collected from the fields. It goes beyond these works by considering the allocation of water among several fields in the same farm, with potentially different crops, also considering the operational limits and costs of irrigation.

3. Problem Description

3.1. SWAMP Pilots

As seen in Table 1, the SWAMP Pilots have different characteristics as well as optimisation objectives. These characteristics and objectives guided the development of two separated approaches for planning: a General approach, for the Guaspari, Intercrop and CBEC pilots; and a Center Pivot-focused approach, aimed at the MATOPIBA pilot.

TABLE 1 SUMMARY OF SWAMP PILOTS

Pilot	Crops and Fields	Irrigation System(s)	Water Supply	Water Availability	Irrigation Objectives
MATOPIBA (Luís Eduardo Magalhães/ Brazil)	Soybeans (main crop); followed by cotton or corn	Center Pivot with Variable Rate Irrigation, covering multiple Management Zones.	Alluvial bank (River)	Not an immediate concern.	Reduce energy costs (pumping of water)
Guaspari (Espírito Santo do Pinhal/ Brazil)	Vineyards	Drip Irrigation	Private water reservoir	Not a concern.	Product quality
Intercrop (Cartagena/ Spain)	Spinach and lettuce	Sprinklers and Drip Irrigation	Private water reservoir (sources include desalinized seawater)	Droughts can affect water availability.	Efficiency of water use and product quality
CBEC (Bologna/ Italy)	Vineyards and pear orchard	Sprinklers and Drip Irrigation	Water needs to be requested to Water Distribution Consortium (CBEC). Delivered through to the farms through channel network.	Supply schedule might impose restrictions on water availability, i.e., not enough water at the ideal time of irrigation.	Reduce water loses in channel network (to be addressed in D3.3)

For the Cartagena and Guaspari pilots the farm is comprised of multiple fields, each growing a single crop and having its own irrigation end system (e.g., valve plus sprinkles) that can be activated independently from the other fields. Differently, in the Italian pilot each farmer typically has a unique field served by a specific irrigation system (sprinkler or drip irrigation). The Water Need models (D3.1) provide the estimates per field. The irrigation plan should provide a schedule that meets the water requirements of the fields while not violating operational constraints, also meeting farmer objectives (profit maximization, reducing water stress, etc.).

In the MATOPIBA case, a single field can be divided into multiple Management Zones (MZ), which is a continuous partition of the field with its own soil properties. The irrigation is done using a Center Pivot system, where a radial array of sprinklers irrigates the field as it moves. In Center Pivot fields, irrigation can be uniform (Uniform Rate Irrigation, URI) or variable (Variable Rate Irrigation, VRI). In URI, the entire field (which can have tens of hectares or more) is irrigated with the same depth, irrespective of Management Zone properties. VRI systems allow for different irrigation depths by region of the field, therefore for a more precise irrigation. VRI can provide the same yield as URI with a reduction of about 30% in water usage (up to 50% depending on the soil type) [29], which also decreases the costs with energy.

However, even with Center Pivots with VRI capabilities, there is a temporal element that needs to be considered. MZs are irrigated as the pivot turns, which takes several hours (at least 12 for high speed systems) or even days to cover the whole field. Time in this case is important not only due to the crop's ideal irrigation timeframe but also energy costs with pumping, which vary with the time of the day in the MATOPIBA region. The role of the Irrigation Planning in this scenario is to find a pivot configuration (depths, irrigation starting time, pivot direction) that addresses the water needs while keeping costs low.

In both cases it is assumed that water can be scarce, i.e., it might not always be possible to meet the water requirements of the crops at the ideal time. Water availability is modelled as a supply schedule or maximum irrigation volume that stipulates the amount of water that can be used during the irrigation window. In case of scarcity, it is the responsibility of the Irrigation Planner to allocate water amongst the fields in a way that best meets farmers' goals.

3.2. Assumptions, Inputs and Outputs

The Water Need models described in Deliverable 3.1 combine knowledge about soil water status (from soil probes), weather conditions and forecast, and crop health (e.g., from drone images) to produce an estimate of how much water the crops will require for the next hours or days. Those models take into account soil water dynamics and plant-soil interactions (e.g., soil moisture at root depth). The Irrigation Planner assumes that the amount estimated, if actually irrigated, will ensure that the crops are not suffering water stress. Conversely, if it is not possible to irrigate the estimated amount, water stress is present and should be taken into account.

From Water Distribution (upcoming Deliverable 3.3), the Irrigation Planner requires a water supply schedule, with restrictions on the amount of water that can be used during the irrigation window (next hours or days). In the case of the pilots that have their own water supplies (e.g., internal reservoirs), the supply schedule becomes the maximum volume to be withdrawn without completely depleting the reservoirs. Figure 2 illustrates how the three modules of Work Package 3 communicate with each other. Table 2 summarizes the inputs the Irrigation Planner takes from each module.

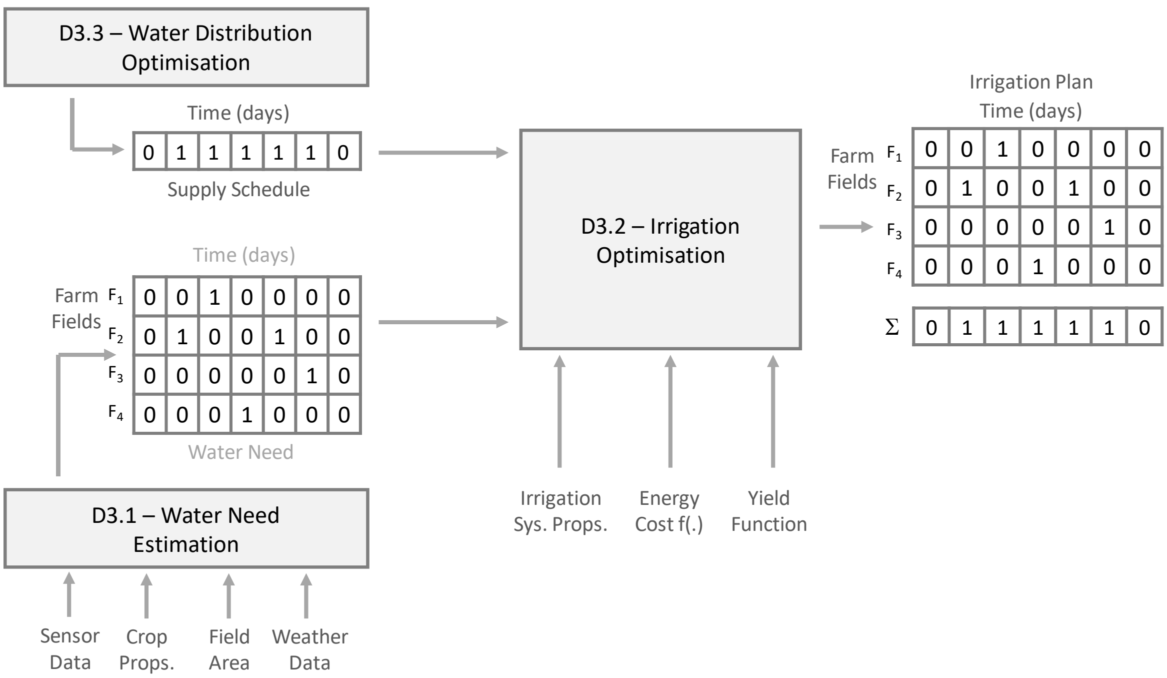

FIGURE 2 RELATIONSHIP WITH OTHER WORK PACKAGE 3 TASKS (SIMPLIFIED).

These general assumptions hold for both the General (Italy, Spain and Guaspari) and Center Pivot (MATOPIBA) scenarios. Differently for the Center Pivot, the Irrigation Planner takes as inputs the Management Zones that comprise the field, described as free-form polygons (geographic coordinates), and their daily water requirements. MZs are mapped by the Planner into discrete regions used by the VRI system (pivot geometry is another input), as shown in Figure 3. As the VRI technology evolves, the resolution (i.e., number of individual regions) also increases. Having MZs as free-form polygons allows for adjusting quickly to new technology. The economic value of the crop in each MZ and its area are used for guiding the allocation of water in case of scarcity.

TABLE 2 IRRIGATION PLANNING INPUTS

Input	Unit	Description
Field (General Scenario, from D3.1)		
Water demand	mm	Water requirements for each field for the next irrigation window.
Field (Center Pivot, from D3.1)		
Water demand	mm	Water requirements for each Management Zone inside the Center Pivot field for the next irrigation window.
Pivot current position	degrees	Azimuth of the pivot at irrigation start.
Farm (from D3.3)		
Water supply schedule	m3/day	Volume available for irrigation for each irrigation day.

The available water supply is one of the constraints the Planner must follow. Another constraint is that the total irrigation amount should not exceed the farm pumping capacity at any given moment. This constraint accounts for real-world limitations of farm's internal water distribution network. The Planner uses energy and cost functions, along with yield productivity and revenue estimates to allocate water and schedule the irrigation events. These constraints and parameters are summarized in Table 3.

TABLE 3 STATIC INFORMATION USED FOR IRRIGATION PLANNING

Properties	Unit	Description
Farm		
Maximum pumping capacity	m3/hour	Maximum water volume pumping per our hour for the entire farm.
Water cost	$/m3	Cost of water itself (when applicable).
Pumping energy consumption	WH/m3	Energy spent with pumping water by volume.
Energy cost	$/WH	Cost of energy by day and hour of the day.
Crop		
Yield per area	Kg/m2	Crop yield in weight by area.
Crop price	$/Kg	Economic return of the yield by weight.
Water Stress Tolerance (Ky)	Ratio	Yield response to water stress (ideally in each stage of development of the crop). Used to estimate losses due to water scarcity.
Irrigation System		
Efficiency	%	Amount of water that effectively reaches the plant roots. The remaining is lost through

		percolation, runoff or evaporation.
Minimum rate	m3/hour	Minimum water volume pumping per our hour.
Maximum rate	m3/hour	Maximum water volume pumping per our hour.
Field (General Case)		
Area	m2	Total area of the field.
Field (Center Pivot)		
Management Zones	Polygons	Coordinates of each Management Zone inside the Center Pivot field.
Geometry	Tracks/ Sectors	Number of tracks (concentric circular paths) and sectors inside each track.

The Irrigation Plan generated at the end contains an hourly schedule of the irrigation events for each field. General statistics about the irrigation are also presented to the farmer, for example: the estimated cost of irrigation (volume of water and energy consumption), and the irrigation error, i.e., how much water could not be delivered to the plants due to scarcity or other constraints. Table 4 details the Irrigation Planner outputs.

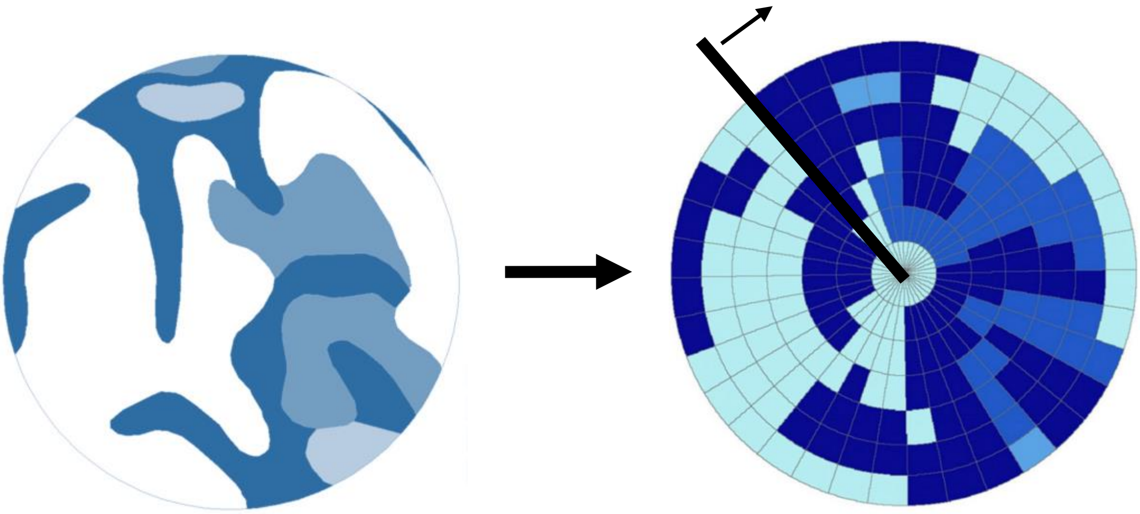

FIGURE 3 PLANNING INPUT (MANAGEMENT ZONES WITH WATER NEEDS) AND OUTPUT (IRRIGATION PER SECTOR) (ADAPTED FROM [19])

As outputs for the Center Pivot scenario, the Irrigation Planner generates a Prescription Map (right side of Figure 3), with an irrigation depth for each sector. The depths will also account for the inefficiency in water application using sprinklers. The direction and time of the day that the pivot must turn to irrigate the sectors is also provided. MZs irrigation demands and energy costs throughout the day direct this choice.

TABLE 4 IRRIGATION PLANNING OUTPUTS

Output	Unit	Description
Irrigation Plan (General Scenario)		
Irrigation depth (hourly)	mm	For each field, specifies the amount of water to be irrigated at each hour.
Irrigation Plan (Center Pivot)		

Irrigation depth (for the cycle)	mm	For each zone of the Center Pivot, the amount of water to be irrigated.
Pivot direction	Clockwise/ Counter-CW	Direction which the pivot will turn to start irrigation.
Pivot start time	Time instant	Time of the day the pivot will start turning and activate the sprinklers.
Irrigation Statistics (field, total)		
Water usage	m3	Volume of water used during the irrigation.
Irrigation cost	$	Cost of irrigation (water and energy).
Water Irrigation cost	$/m3	Cost by volume of water used.
Irrigation error	%	Difference from water requirements and actual irrigation (discounting inherent loses from irrigation systems).
Irrigation efficiency	%	Volume that reached the plants over total volume used.

4. Modelling and Solutions

This section describes the Irrigation Planning problem formally, first in general terms in section 4.1, and then as a Linear Programming problem in 4.2. Section 4.3 formalizes the Center Pivot special case, also including an initial nonlinear programming formulation. Metaheuristic alternatives are discussed in section 4.4. Section 4.5 proposes the use of Machine Learning for improving planning through more precise estimates of water efficiency.

4.1. General Formulation

The irrigation scheduling problem consists in finding an irrigation plan, $Plan$, that meets an optimisation objective (e.g., maximize profit) while not violating the constraints (e.g., total available water, maximum water stress). The irrigation plan, $Plan = \{Depth_{ft}\}$, $f = 1..N$, $t = 1..T$, describes the irrigation depth for field f in time instant t, with N being the number of fields and T the number of time steps (e.g., hours) of the irrigation window. Table 5 contains an example of irrigation plan.

TABLE 5 EXAMPLE OF AN IRRIGATION PLAN.

$Depth_{ft}$	$t = 1$	$t = 2$	$t = 3$	…	$t = T$
$f = 1$	2 mm	0	5 mm	…	3 mm
$f = 2$	0	0	0	…	4 mm
$f = 3$	1 mm	3 mm	0	…	2 mm
…	…	…	…	…	…
$f = N$	4 mm	0	0	…	0

For the MATOPIBA scenario, for example, a typical objective is to find a plan that offers maximum profit, $Profit = R - C$, where:

$$R = \sum_{f} R_f \tag{1}$$

$$C = \sum_{f} C_f \quad (2)$$

R is the expected revenue from the crop and C, the cost of irrigation. R_f and C_f are the revenue and cost for a given field f, respectively, as defined below:

$$R_f = Y_f * A_f * P_f \quad (3)$$

$$C_f = WC_f + EC_f \quad (4)$$

P_f is the price of crop by weight, A_f is the area of field f, and Y_f is the yield (by area) obtained with the irrigation plan. WC_f is the cost of water and EC_f, the cost with energy.

The yield of a crop is a function the water stress it was submitted. Different crops deal with water stress differently. The tolerance to stress is given by the factor K_y, which relates the actual and maximum yields to the water stress. K_y is defined according to the formulation in (5), extracted from FAO Water Reports 22 [30]. K_y values for different crops can be found in FAO manuals and the literature.

$$\left(1 - \frac{Y_a}{Y_m}\right) = K_y\left(1 - \frac{ET_a}{ET_m}\right) \quad (5)$$

Y_a is the actual yield (by area) and Y_m the maximum yield for a given crop. ET_a and ET_m are the actual and ideal evapotranspirations. To calculate the yield resulting from irrigation for a field f, Y_f, the evapotranspiration measures ET_a and ET_m are replaced by the (effective) irrigated volume $W_f = Depth_f * A_f$ and the water demand in volume $D_f = WaterNeed_f * A_f$, respectively:

$$\left(1 - \frac{Y_f}{Y_f^{max}}\right) = K_{yf}\left(1 - \frac{W_f}{D_f}\right) \quad (6)$$

$WaterNeed_f$ is the demand estimated by the Water Need models in mm. By developing (6), the formulation below is obtained:

$$Y_f = Y_f^{max} - Y_f^{max}K_{yf} + Y_f^{max}K_{yf}\frac{W_f}{D_f} \quad (7)$$

In this equation, $Y_f^{max} - Y_f^{max}K_{yf}$ is the baseline yield (i.e., the yield if no irrigation is applied in field f), while $Y_f^{max}K_{yf}\frac{W_f}{D_f}$ is the yield gain with irrigation. By multiplying these factors by the area of the field A_f and price of the crop P_f, the baseline revenue $\bar{R}_f$ and the revenue gain with irrigation $\hat{R}_f$ are obtained:

$$\bar{R}_f = P_f A_f Y_f^{max} - P_f A_f Y_f^{max} K_{yf} \quad (8)$$

$$\hat{R}_f = \frac{P_f A_f Y_f^{max} K_{yf}}{D_f} \quad (9)$$

For the cost part of the $Profit$ formulation, the cost of water is given by $WC_f = W_f * WC$, with WC being the price for the cubic meter (m^3) of water provided by the farmer. The expenses with energy, EC_f, are based on the cost of pumping the water to the field f:

$$EC_f = \sum_{t}^{T} W_{ft} * PEC * EC_t \quad (10)$$

PEC is the energy consumption for pumping x cubic meters by unit of time. EC_t gives the cost of energy at time t, and is used to allow for different tariffs depending on the time of the day.

4.2. Modelling the General Case with Linear Programming

Linear programming (LP) [17] is a popular optimisation technique, often used in scheduling problems, having been applied for irrigation planning in the literature (for example, Reca et al. in [24]). It has the advantage of finding an optimal solution for the problem (if it exists), and various software packages that solve LP problems are available, both free (e.g., GLPK[1]) and paid (e.g., IBM CPLEX[2]). In order to achieve linearity, however, frequently the problem needs to be simplified by relaxing some constraints, which can affect the applicability of the solutions found. In this section, the SWAMP's irrigation planning problem is modelled using LP, employing the general formulations described in the previous section. The parameters and notation are summarized in Table 6.

TABLE 6 OPTIMISATION PARAMETERS FOR THE GENERAL SCENARIO.

T	Number of time steps in the irrigation window
F	Number of fields
E_f	Irrigation efficiency of the irrigation system in field f (%)
D_f	Water demand (volume) for field f ($D_f = WaterNeed_f * Area_f$)
$\hat{R}_{ft}$	Revenue gain by water volume for field f at time t
PEC	Pumping energy consumption (by water volume)
EC_t	Energy cost at time t
WC_f	Water cost in field f (by volume)
$Volume_{max}$	Maximum total volume of irrigation
$Water_{min}$	Minimum (under) irrigation (percentage of demand, e.g., 90%)
$Water_{max}$	Maximum (over) irrigation (percentage of demand, e.g., 120%)
$Cost_{max}$	Maximum total cost of irrigation
$Pumping_{max}$	Maximum pumping volume per time unit

The decision variables (i.e., the irrigation plan), for the linear program are:

W_{ft} : volume of water irrigated in field f in period t ($W_f = Depth_f * A_f$);

When optimizing for the total profit, for example, the optimisation objective becomes:

$$Maximize\ Profit = \sum_{f}^{F} \left[\sum_{t}^{T} W_{ft} * E_f * \hat{R}_{ft} - W_{ft} * WC_f - W_{ft} * PEC * EC_t \right] + \bar{R}_f \tag{11}$$

This objective is subject to the following constraints**:**

- Every field f should be irrigated within the allowed range:

$$\sum_{t=1}^{T} E_f * W_{ft} \leq D_f * Water_{max} \qquad , \forall f \tag{12}$$

$$\sum_{t=1}^{T} E_f * W_{ft} \geq D_f * Water_{min} \qquad , \forall f \tag{13}$$

- The pumping capacity of the farm should not be exceeded for any moment t.

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[1] https://www.gnu.org/software/glpk/

[2] https://www.ibm.com/br-pt/analytics/cplex-optimizer

$$\sum_{f=1}^{F} W_{ft} \leq Pumping_{max} \qquad ,\forall\, t \tag{14}$$

- The total volume consumed should not exceed to allotted amount $Volume_{max}$for the irrigation window.

$$\sum_{f=1}^{F}\sum_{t=1}^{T} W_{ft} \leq \; Volume_{max} \tag{15}$$

- The total cost should be limited by $Cost_{max}$:

$$\sum_{f=1}^{F}\sum_{t=1}^{T} W_{ft} * WC_f + W_{ft} * PEC * EC_t \leq Cost_{max} \tag{16}$$

- General constraints on the decision variables:

$$W_{ft} \geq 0 \tag{17}$$

Other optimisation objectives from the pilots can be modelled in a similar way. This modelling, however, has its limitations, intended to keep it linearly solvable. For example, it treats revenue as a linear function of the water volume (i.e., more water always means more revenue), while overirrigation is known to cause losses in yield. This incentivises the planner to allocate more water than necessary. To counter this effect, constraint (12) limits the amount of water used per field relative to its demand. Constraints (14) and (15), in pumping capacity and total volume, also limit excessive watering. When water is scarce, however, constraints (15) and (13) (minimum irrigation) can make it impossible to find a plan. In this situation, the farmer will have to relax or remove constraint (13) (e.g., $Water_{min} \; = \; 0$) to obtain a plan.

This model also does not consider the time dependency between irrigation events, i.e., the fact that irrigating at time t affects the soil water content at time $t + 1$. It also tends to spread the irrigation through time in a manner that might not be feasible in practice (especially in a field manually operated). This can be avoided by adding a constraint that limits the operation times.

4.3. Modelling the Center Pivot Scenario

In the Center Pivot (CP) case, the field is organized as a set of concentric tracks and radial sectors, with regions (c, s) defined by track c and sector s. The geometry of the field (i.e., the number of tracks and sectors) is pre-defined during platform configuration. The Management Zones are regions inside the CP field that have similar soil characteristics and therefore can be managed in the same way. The division in sub-fields with same properties allows for fewer soil probes, since ideally only one is necessary per MZ. Management Zones and the mapping to the Center Pivot is shown in Figure 4.

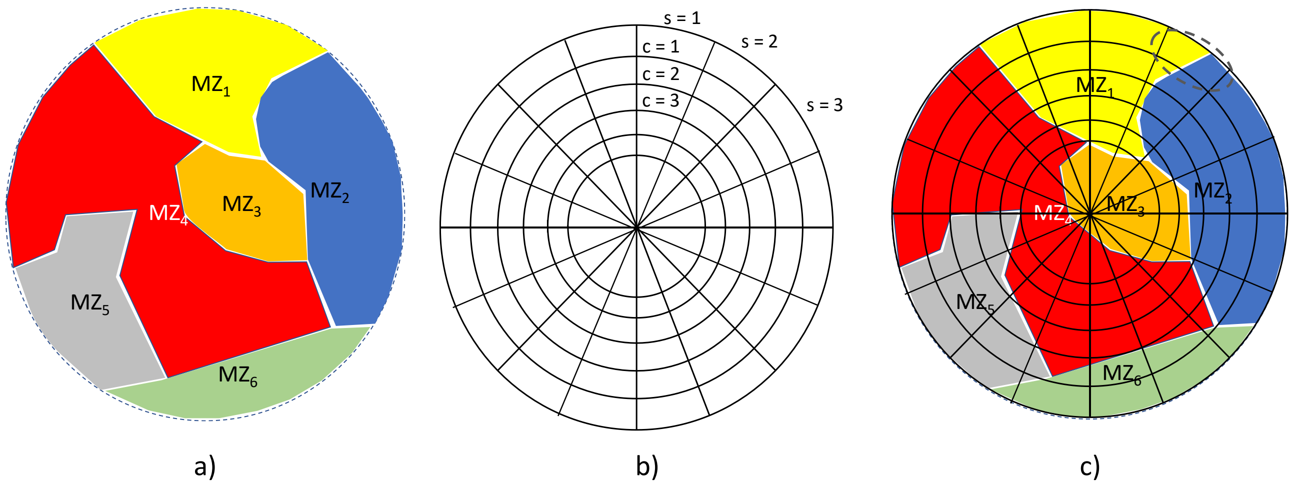

FIGURE 4 A) MANAGEMENTS ZONES INSIDE THE CENTER PIVOT FIELD. B) CENTER PIVOT GEOMETRY (TRACKS AND SECTORS). C) SELECTED REGION (1,2) IS COMPOSED (APPROX.) OF 50% MZ1 AND 50% MZ2.

In this scenario, the irrigation plan becomes a prescription map, $Map = \{Depth_{cs}\}$, which contains the irrigation depth at each region delimited by track c and sector s. Before planning starts, however, it is necessary to map the Management Zones contained in the Center Pivot field into the tracks and sectors. The process consists in finding how much of each Management Zone intersects with each (track, sector) of the CP field. The Python library Shapely[3], a "package for manipulation and analysis of planar geometric objects", can be used to solve that. Another method is using a randomized approximation, where points are randomly spread over the CP field. The map $M_z(c,s)$ between Management Zones and regions (c, s) is determined by the proportion of points that fall into that MZ and (c, s). Since this calculation is done only once for the entire irrigation season, its computational cost should not be a major concern. Table 7 shows an example of mapping function $M_z(c,s)$.

TABLE 7 EXAMPLE OF WEIGHTS MAPPING MANAGEMENT ZONES INTO (TRACKS, SECTORS)

$M_z(c,s)$	$z = 1$	$z = 2$	$z = 3$	…	$z = N$
(1,1)	0.5	0.5	0	…	0
(1,2)	0	0	1.0	…	0
(1,3)	0	1.0	0	…	0
…	…	…	…	…	…
(2,0)	0.3	0	0.7	…	0
(2,1)	0	0.8	0.1	…	0.1
…	…	…	…	…	…

The weights in Table 7 are used for adjusting to Tracks and Sectors the estimates provided originally for Management Zones. For example, being $D_z = WaterNeed_z * A_f$ the estimated water demand for the Management Zone z, in volume, the demand for each region (c,s) of the field, D_{cs}, is calculated by:

$$D_{cs} = \sum_z M_z(c,s) * D_z \tag{18}$$

[3] Shapely: https://pypi.org/project/Shapely/

The same process can be used to calculate the irrigation volume applied to the zone z, W_z, from the volume applied in the regions (c, s), W_{cs}, and other estimates necessary for the optimisation process.

Differently from the Irrigation Plan described in previous sections, the timing of irrigation of the regions is determined implicitly by the starting time (T_{start}), direction (Dir) and (angular) speed of the Center Pivot. Pivot speed is considered a parameter of the system, while starting time and direction are decision variables computed by the Irrigation Planner along with the amount of water per region (c, s), W_{cs}. Regions (c, s) in the same sector s are irrigated at the same time. Other parameters in this scenario are shown in Table 8.

TABLE 8 OPTIMISATION PARAMETERS FOR THE CENTER PIVOT SCENARIO.

T	Number of time steps in the irrigation window
Z	Number of management zones
C	Number of tracks in the Center Pivot field
S	Number of sectors in the Center Pivot field
E	Water efficiency of the Center Pivot irrigation system (%)
D_z	Water demand (m^3) for Management Zone z ($D_z = WaterNeed_z * A_z$)
D_{cs}	Water demand (m^3) for region (c, s) of the Center Pivot field
$\hat{R}_{cs}$	Revenue gain by water volume for region (c, s)
PEC	Pumping energy consumption (by water volume)
$Volume_{max}$	Maximum total volume of irrigation
$Water_{min}$	Minimum (under) irrigation (percentage of demand, e.g., 90%)
$Water_{max}$	Maximum (over) irrigation (percentage of demand, e.g., 120%)
$Cost_{max}$	Maximum total cost of irrigation
$Pumping_{max}$	Maximum pumping volume per time unit

Similar to the general case described in section 4.2, using decision variables T_{start}, Dir, and W_{cs}, the optimisation problem for the Center Pivot scenario can be stated as:

$$Maximize\ Profit = \sum_{c}^{C} \sum_{s}^{S} W_{cs} * E * \hat{R}_{cs} - W_{cs} * WC - W_{cs} * PEC * EC_s + \bar{R}_{cs} \tag{19}$$

Subject to:

- Every region (c, s) should be irrigated within the allowed range:

$$\sum_{t=1}^{T} E * W_{cs} \leq D_{cs} * Water_{max} \qquad , \forall z \tag{20}$$

$$\sum_{t=1}^{T} E * W_{cs} \geq D_{cs} * Water_{min} \qquad , \forall z \tag{21}$$

- The pumping capacity of the farm should not be exceeded for any moment t. (function $h_s(.)$ tells if sector s was irrigated at time t, given T_{start} and Dir.)

$$\sum_{c=1}^{C}\sum_{s=1}^{S} W_{cs} * h_s(t, T_{start}, Dir) \leq \; Pumping_{max} \qquad , \forall\, t \tag{22}$$

- The total volume consumed should not exceed to allotted amount $Volume_{max}$ for the irrigation window.

$$\sum_{c=1}^{C}\sum_{s=1}^{S} W_{cs} \leq \; Volume_{max} \tag{23}$$

- The total cost should be limited by $Cost_{max}$:

$$\sum_{c=1}^{C}\sum_{s=1}^{S} W_{cs} * WC + W_{cs} * PEC * EC_s \leq Cost_{max} \tag{24}$$

- General constraints on the decision variables:

$$W_{cs} \geq 0 \tag{25}$$

$$0 \leq T_{start} \leq \mathrm{T} \tag{26}$$

$$Dir = \{0{,}1\} \tag{27}$$

The optimisation problem above, however, is not linear. EC_s is the energy cost for sector s, and is a function of EC_t (cost of energy at time t), the start time T_{start} and direction Dir, i.e., $EC_s = h(EC_t, T_{start}, Dir)$. (The function $h(.)$ calculates the energy cost of irrigating sector s based on the time the pivot crosses it). Even $h(.)$ being a linear function, the final function will have decision variables multiplying each other, both in the optimisation function and the constraints (e.g., the cost component is given by $W_{cs} * PEC * EC_s$). In such cases, the optimisation problem becomes a Quadratically Constrained Quadratic Programming (QCQP) problem, which in the general case is NP-hard.

One way to handle the complexity is to work with discrete starting times, $T_{start} \in \{1, 2, .., T\}$, and solve the linear version of the problem for each scenario (and both directions), selecting the best solution at the end. For a 24 hours irrigation window, 1-hour time steps and 12 hours for a complete turn of the pivot, and assuming that the pivot should complete its turn within the irrigation window T, 24 total scenarios need to be tested (12 starting times for each direction). Another approach is to relax the constraints and optimisation function to obtain a linear model. In both approaches, numerical experiments are necessary to determine if the plans generated are appropriate and can be computed in adequate time.

4.4. Using Metaheuristics for Irrigation Planning

The irrigation scheduling problem could be classified as NP-Hard problem [2], [8]. Consequently, due to the computational complexity, in some cases, it not feasible to find the optimal solution due to the large space of possible solutions. Even exact methods, such as linear programming, often also fail to find a solution due to the lack of scalability of this technique (high computational resource demand).

To overcome the limitations of exact methods, it is common to use metaheuristics to find an optimal or quasi-optimal solution. A metaheuristic is a computational procedure for finding an approximated and acceptable solution for a problem in less time and with less computational effort when compared with exact methods. They sample iteratively the search space, guiding the process towards the best solutions using some quality measure. They also make few assumptions about the underlying problem, making them very flexible.

In the literature it is possible to find some examples of metaheuristics to solve Irrigation Planning Problems: Genetic Algorithms in [2], [22], Simulated Annealing in [6], Ant Colony Optimisation in [21], Swarm

Intelligence [8], among others. This section presents how SWAMP's irrigation planning problem can be solved using two metaheuristics: Genetic Algorithms and GRASP.

4.4.1. Genetic Algorithms

Genetic Algorithms [4] are a well-known metaheuristic inspired by the process of natural selection. The basic idea consists of, starting from an initial pool of random solutions (chromosomes), progressively selecting and combining the most promising ones, until a certain threshold is met (in quality or number of iterations). The final solution will be the best chromosome in the pool (or population) in the last iteration (generation). The quality of a solution is measured by a fitness function derived from the problem. Figure 5 depicts the typical stages of a GA:

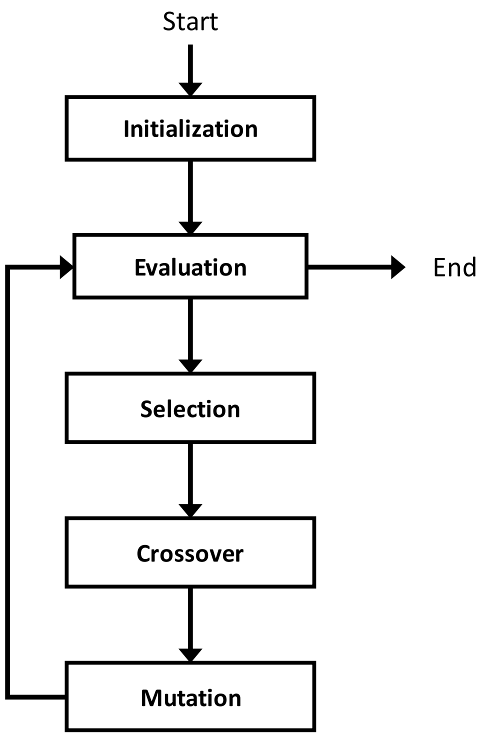

FIGURE 5 GENERAL OUTLINE OF A GENETIC ALGORITHM.

In the Irrigation Schedule problem, chromosomes are the plans (or prescription maps), and their fitness is determined by the target optimisation function, for example, the Profit of irrigation as calculated in section 4.1. The stages in Figure 5 are described below for the irrigation planning problem:

- **Initialization:** a number of plans are generated by assigning randomly generated values to the genes, in this case, the irrigation volumes W_{ft}. Problem constraints, such as maximum volume $Volume_{max}$, can be used to generated values in the solution range. Invalid plans are not included in the population. The number of chromosomes in the population is a parameter that needs to be fine-tuned.
- **Evaluation:** based on the fitness of the current population and the number of iterations, the algorithm decides if it should continue or not.
- **Selection:** plans are selected to be combined based on their fitness, i.e., how profitable a plan is. Random selection of chromosomes where the probability is determined by the fitness value is a common strategy.
- **Crossover:** the next generation is created by combining the selected plans pair-wise, hoping that the resulting plan inherits the best features of both. This can be done in several ways. In a field-wise way, the new plan will inherit schedules for some fields from one parent, and the remaining schedules from the other. The same can be done in a time-wise way. Another approach is to

average the individual genes (i.e., irrigation volumes) from the parents (parent fitness can be used to weight the average). These different approaches need to be compared through experimentation.

- **Mutation:** new plans are subject to random changes in order to allow the exploration of the solution space. For the planning case, genes are the irrigation volumes in time (t) and space (f), W_{ft}, and the mutation consists of adjusting the value up or down with a given probability. The level of adjustment can be precomputed from the demands, D_{ft}, in order to guide the search towards more reasonable values.

With Genetic Algorithms, as with other metaheuristics, the quality of the solution (i.e., how close to the actual optimum) will depend on the time it is allowed to run. In this case, the time is determined by the maximum number of population generations. Experiments are necessary to determine a reasonable number of generations that approximates the optimal solution well enough (e.g., optimisation error of 5% or less) within practical time constraints (in this case, how long the farmer is willing to wait for the plan to be computed). The quality and time necessary to generate a solution also depend on other parameters such as population size, crossover and mutation thresholds, etc. Although the literature on GA offers guidance on the selection of these parameters, they will need to be fine-tuned experimentally.

4.4.2. GRASP

The Greedy Randomized Adaptive Search Procedure (GRASP) is a general-purpose metaheuristic based semi-greedy solution construction. This step is followed by a local search to improve the solution. The general steps of GRASP are shown in Figure 6.

```
procedure GRASP(Max_Iterations,Seed)
    Read_Input();
    for k = 1,...,Max_Iterations do
        Solution ← Greedy_Randomized_Construction(Seed);
        Solution ← Local_Search(Solution);
        Update_Solution(Solution,Best_Solution);
    end;
    return Best_Solution;
end GRASP.
```

FIGURE 6. PSEUDO-CODE OF THE GRASP METAHEURISTIC [11]

The initial stage creates a solution using a semi-greedy search that adds elements to the solution, for example, depth values to an irrigation plan. A random selection is performed on a restricted set of elements ranked according to some greedy quality function, which form the Restricted Candidate List, RCL. In the planning problem, these elements can be irrigation depths at certain fields ranked according to the revenue gain from irrigation. Figure 7 contains the pseudo-code for the construction phase [11].

```
procedure Greedy_Randomized_Construction(Seed)
    Solution ← ∅;
    Evaluate the incremental costs of the candidate elements;
    while Solution is not a complete solution do
        Build the restricted candidate list (RCL);
        Select an element s from the RCL at random;
        Solution ← Solution ∪ {s};
        Reevaluate the incremental costs;
    end;
    return Solution;
end Greedy_Randomized_Construction.
```

FIGURE 7. PSEUDO-CODE OF THE CONSTRUCTION PHASE [11]

After the generation of a candidate solution, a local search is then performed (Figure 8). The local search is not particularly specified by GRASP. There are several local search strategies that can be implemented in this step, for example, Variable Neighbourhood Search (VNS) [20] or tabu search [12]. The objective of this step is to cause small perturbations in the solution to find similar, hopefully better, solutions. In practice, this step would cause small changes in the allocated irrigation depths in the plan.

```
procedure Local_Search(Solution)
    while Solution is not locally optimal do
        Find s' ∈ N(Solution) with f(s') < f(Solution);
        Solution ← s';
    end;
    return Solution;
end Local_Search.
```

FIGURE 8 PSEUDO-CODE OF THE LOCAL SEARCH PHASE [11]

At the end of each iteration the new solution is compared to the best solution so far. If it is better, it becomes the best solution. Otherwise, it is discarded. The number of iterations is determined by the user, and therefore must be tested empirically. The GRASP metaheuristic is easy to implement, even taking into consideration complex variables and models. The memory consumption is not usually a problem since one solution is evaluated at a time, as opposed to Genetic Algorithms where a large population of solutions exist at any time of the process. One the other hand, GA have the advantage of combining promising solutions through crossover, which can improve their quality and time taken to compute them.

4.5. Improving Planning with Machine Learning

As described in the previous sections, the efficiency of the irrigation system (E_f, the percentage of water that actually reaches the plants) is one of the parameters used by the planner in order to calculate how much to irrigation and when. So far, predefined values taken from the literature were assumed, for example, those from FAO recommendations in Table 9 below.

TABLE 9 EFFICIENCY OF IRRIGATION METHODS, ADAPTED FROM [5], ANNEX I, TABLE 8.

Irrigation methods	Field application efficiency
Surface irrigation (border, furrow, basin)	60%
Sprinkler irrigation	75%
Drip irrigation	90%

These values were good enough for the coarse-grained irrigation scheduling applied traditionally. However, each irrigation system installation has its peculiarities, and it is expected that their efficiency will be affected by the geography of the farm, weather and time of water application. The availability of IoT devices, including soils sensors and water meters, in combination with data-driven techniques, can be used to fill this gap. Figure 9 shows the updated data flow between the SWAMP irrigation services, including a component responsible for calculating the actual water efficiency for the farm's irrigation system.

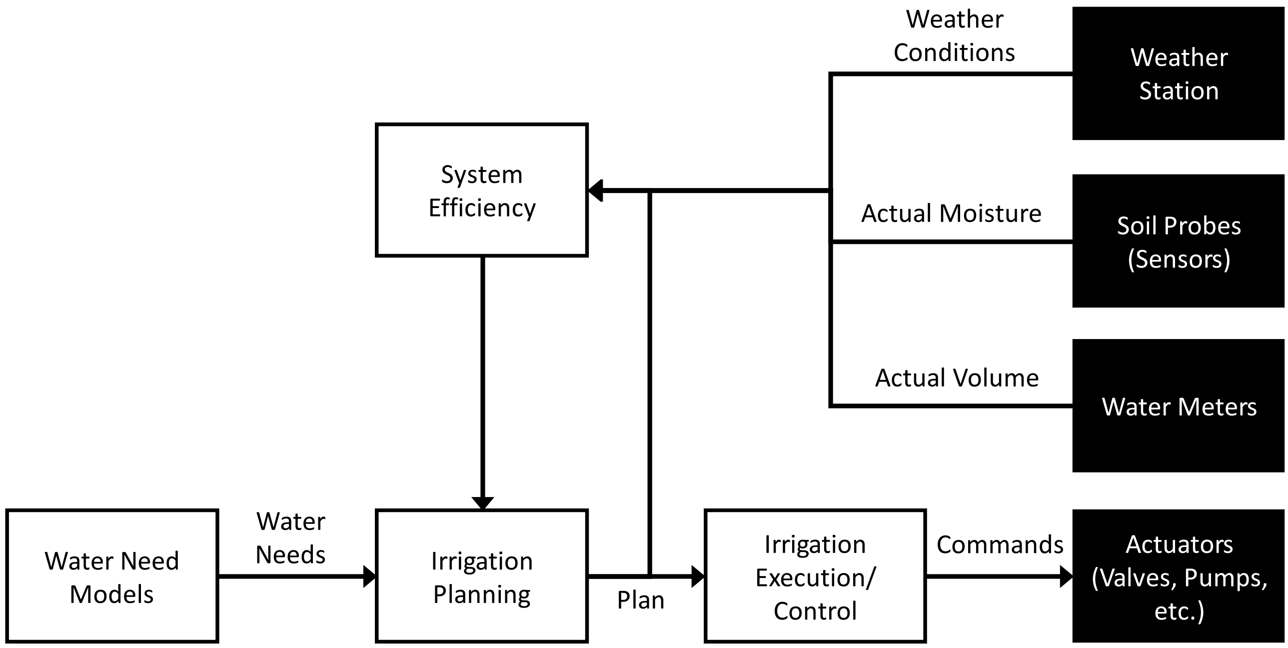

FIGURE 9 UPDATED GENERAL DATA FLOW BETWEEN IRRIGATION SERVICES, ADDING SYSTEM EFFICIENCY COMPONENT.

In order to create the System Efficiency box, the CRISP-DM methodology can be applied [26], with its six phases: Business Understanding, Data Understanding, Data Preparation, Modelling, Evaluation, and Deployment. Techniques from simple linear regression to more advanced regression trees (e.g., XGBoost [7]) can be applied to historical data to generate an efficiency model, which is then implemented in the System Efficiency box.

This strategy, however, has the disadvantage that changes in the farm's irrigation infrastructure can automatically invalidate the model, which will need to be recreated by a data analyst. An alternative approach is to implement an online modelling technique, which runs continuously in the platform. In this case, the System Efficiency module would automatically adjust the model to the new data as the it deviates from the actual measurements. One of such approaches for online regression models is described in [13]. The disadvantage in this case is that it takes the data analyst out of the loop, which might affect the final quality of the model.

5. Discussion and Next Steps

The strategies described in this document make few assumptions about the input provided by water need models. However, as the work in these models progresses based on the ideas described in Deliverable 3.1, improvements can also be made to the planning process as well. For example, guidance on the precise timing of irrigation from Water Need models can narrow down the search space and improve the solutions.

About the use of metaheuristics (section 4.4), they enable the use of arbitrary objective functions, which can include sophisticated simulations of the soil-plant-air system's response to irrigation at specific times, for example. This approach, however, would have to leverage the computing power of the cloud to offer low response times for the users.

Finally, the selection of an optimisation approach depends on the requirements imposed by the problem. The strategies in this document were conceived based on the current understanding of the planning problem as applied to the SWAMP scenarios. Some simplifications were made to keep the models solvable, which need to be verified numerically and in the field. Field experiments, in special, will be critical to determine the feasibility of the plans as well as to validate the initial requirements, since often during these tests new requirements emerge. Changes in requirements, however, can force the optimisation solution to be completely redone. With that in mind, a two-pronged action plan is proposed:

- Implement and test a basic version (e.g., greedy) of the irrigation planner and the other services in WP3. Implementation will be iterative and with focus on validating assumptions, dataflow between services, and consolidation of requirements from the pilots.
- In parallel, evaluate and validate numerically the solutions described in this document, starting from the simpler ones (linear models for the general case, GRASP for the Center Pivot). These solutions should be confronted and adapted to changes in requirements as they happen. The solutions with best numerical performance will be put to test in the field. New solutions not described in this document can be proposed and tested during this process.

6. Conclusions

The scheduling of irrigation events is a long-lasting challenge in agriculture. Traditional scheduling methods rely on the historical statistics and general guidelines to provide the farmer with a plan that can be put in practice, usually for the entire irrigation season. Last decades saw the rise of computational methods for solving the scheduling problem with more accuracy, at the price of more complex plans. The recent emergence of Internet of Things (IoT) technologies brings real-time situation awareness of the farm into irrigation planning. It also enables automation of irrigation activities, which allows for more sophisticated schedules to be put in place.

This deliverable described the Irrigation Planning problem in the SWAMP scenario: its assumptions, objectives, and constraints. It also proposed strategies to address the problem by employing consolidated optimisation methods. The next step will be the numerical experimentation validation and comparison of these approaches, both regarding the quality of the solutions and the performance of the algorithms. However, SWAMP being a pilot-driven project, any final validation will have to come from the fields.

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